Qualifying Exam Syllabus for Dmitri Pavlov (pavlov@math)

Date: August 25, 2008

Major topic: Algebraic topology (Geometry)

Ordinary homology and cohomology: Standard exact sequences, cup and cap products, universal coefficients, Künneth formula, transfer homomorphism, local coefficients, Poincaré duality, Alexander duality, Bockstein homomorphisms for $\mathbb{Z}/(p)$, Lefschetz fixed point theorem, H-spaces, Hopf algebras, and cohomology of SO.

Homotopy: Whitehead theorem, Eilenberg-Mac Lane spaces, Hurewicz theorem, CW approximations, excision (with additional conditions) for homotopy groups and Freudenthal suspension theorem, fibrations, Whitehead products and stable homotopy groups, Postnikov tower, obstruction theory, loopspace and suspension functors, Leray-Hirsch theorem, cohomology operations, Steenrod squares and powers, cohomology of U, SU, and Sp, cohomology of Grassmannians, Gysin sequence, Thom class, Thom space, and Thom isomorphism, Eckmann-Hilton duality, Dold-Thom theorem.


Major topic: Complex geometry (Geometry)

Basic definitions, divisors and line bundles.
Kähler identities, Hodge decomposition, Hard Lefschetz theorem, Hodge conjecture.
Serre duality, connections, curvature, Chern classes and Chern-Weil theory.
Kodaira vanishing theorem, Kodaira embedding theorem.
Deformation of complex structures, Kodaira-Spencer theorem, Kuranishi theorem. (Without proofs.)

Minor topic: von Neumann algebras (Modern analysis)

GNS construction.
Definition of von Neumann algebras.
von Neumann bicommutant theorem.
Kaplansky density theorem.
Commutative von Neumann algebras.
Geometry of projections in a factor and types of factors.
The trace and the standard form of type $\text{II}_1$ factor.

References