COHOMOLOGY OF TOPOLOGICAL GROUPS (*)

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The following work was done jointly by G. J. Mitchison and me. Its purpose is to define cohomology groups of a topological group G with coefficients in a topological abelian group A (on which G acts continuously) by a method analogous to defining the cohomology of a discrete group as a derived functor. I shall begin by introducing a category of topological abelian groups.

§ **1.** Let *Topab* denote the category of compactly generated and locally contractible (1) hausdorff topological abelian groups and continuous homomorphisms. A sequence $A' \xrightarrow{i} A \xrightarrow{p} A''$ in *Topab* will be called a *short exact sequence* if

i) i is an imbedding of A' as a closed subgroup of A;

ii) p induces a topological isomorphism $A/A' \xrightarrow{\cong} A''$;

iii) A' has a local cross-section in A (i.e. the fibration $A' \to A \to A''$ is locally trivial topologically).

From short exact sequences one can build up longer exact sequences in the usual way.

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(1) I mean an abelian group A in the category of k-spaces [2]. Thus the composition $A \times A \to A$ is supposed to be continuous when the product is understood in the sense of k-spaces. The condition of local contractibility is inserted only to ensure that each object has a contractible resolution ((1.1) below); in fact *Topab* can be embedded in a larger abelian category with enough injectives, and the restriction is unnecessary.

PROPOSITION 1.1. Any object A of *Topab* has a contractible resolution, i.e. there is an exact sequence

$$0 \to A \to A^0 \to A^1 \to A^2 \to \dots$$

such that each A^i is contractible as a topological space.

PROOF. A suitable resolution is, in the notation of the appendix:

$$A \to EA \to EBA \to EB^2A \to \dots$$

A left-exact additive functor $F: Topab \to Ab$, where Ab is the category of abelian groups, will be called *derivable* if, for each short exact sequence $A' \to A \to A''$ in *Topab* for which A' is contractible, the sequence

$$0 \to F(A') \to F(A) \to F(A'') \to 0$$

is exact. If F is such a functor one can define its derived functors R^pF for $p \geq 0$ by $R^pF(A) = H^p(F(A^\cdot))$, where $A \to A^\cdot$ is a contractible resolution of A. This is justified by

PROPOSITION 1.2. If F is derivable then $R^pF(A)$ does not depend on $A^\cdot$.

PROOF: Let $E^\cdot_A$ be the canonical contractible resolution constructed above. Consider the double complex

$$\begin{array}{ccccccc} A & \longrightarrow & A^0 & \longrightarrow & A^1 & \longrightarrow & \dots \\ \downarrow & & \downarrow & & \downarrow & & \\ E^\cdot_A & \longrightarrow & E^\cdot_{A^0} & \longrightarrow & E^0_{A^1} & \longrightarrow & \dots, \end{array}$$

in which both the rows and the columns are exact (because the functors E and B are exact, as explained in the appendix), and all the objects except those in the first row are contractible. When F is applied to this complex the rows and the columns, except for the first in each case, remain acyclic because F is derivable, so the cohomology of the first row is isomorphic to that of the first column, i.e. $R^pF(A)$ defined using $A \to A^0 \to A^1 \to \dots$ is the same as $R^pF(A)$ defined using the canonical resolution $A \to E^\cdot_A$.

Given the exactness of $A \mapsto E^\cdot_A$, it is clear that we have

PROPOSITION 1.3. If $F : Topab \to Ab$ is derivable, and $A' \to A \to A''$ is a short exact sequence in *Topab*, then there is an infinite exact sequence

$$0 \to F(A') \to F(A) \to F(A'') \to$$
$$\to R^1F(A') \to R^1F(A) \to R^1F(A'') \to R^2F(A') \to \dots$$

EXAMPLE 1.4. Let X be a paracompact space. Then the functor $A \mapsto \mathrm{Map}(X; A)$, where $Map(X; A)$ denotes the space of continuous maps of X into A, is derivable, and $R^p \,\mathrm{Map}(X; A) \cong H^p(X, \mathbf{A})$, the cohomology of X with coefficients in the sheaf of germs of continuous functions from X into A.

PROOF: If $A' \to A \to A''$ is a short exact sequence in *Topab* with A' contractible then $A \cong A' \times A''$ as topological space, for a locally trivial principal fibration with a contractible fibre is trivial (2). So $\mathrm{Map}(X; A) \to \mathrm{Map}(X; A'')$ is surjective, and the functor is derivable.

From the short exact sequence $A \to EA \to BA$ one obtains an exact sequence

$$\mathrm{Map}(X; EA) \to \mathrm{Map}(X; BA) \to R^1 \,\mathrm{Map}(X; A) \to 0.$$

This shows that $R^1 \,\mathrm{Map}(X; A) \cong [X; BA]$, the set of homotopy-classes of maps from X to BA, for a map $X \to BA$ is null-homotopic if and only if it lifts to EA. But BA is a classifying-space for locally trivial principal A-bundles on paracompact spaces, so

$$[X; BA] \cong H^1(X; \mathbf{A}),$$

and

$$R^1 \,\mathrm{Map}(X; A) \cong H^1(X; \mathbf{A}).$$

Finally, one sees that $R^p \,\mathrm{Map}(X; A) \cong H^p(X; \mathbf{A})$ for all A by induction on p. For $R^p \,\mathrm{Map}(X; A) \cong R^{p-1} \,\mathrm{Map}(X; BA)$ if $p \geqslant 2$; and $H^p(X; \mathbf{A}) \cong H^{p-1}(X; \mathbf{BA})$ if $p \geqslant 2$ because $\mathbf{EA}$ is a soft sheaf [3].

§ 2. Let G be a compactly generated topological group, and let *G-Topab* be the category of compactly generated locally contractible topological abelian groups A on which G acts continuously. A sequence

(2) See Appendix B.

$A' \to A \to A''$ in *G-Topab* will be a short exact sequence if it is one in *Topab* when the *G*-actions are neglected.

Call an object of *G-Topab soft* if it is of the form $Map(G;\ A)$, where A is a contractible group and *Map* denotes the set of continuous maps with the compact-open topology. (Thus a soft group is contractible). And call a left-exact additive functor $F: Topab \to Ab$ *derivable* if for each short exact sequence $A' \to A \to A''$ with A' soft the sequence $0 \to F(A') \to F(A) \to F(A'') \to 0$ is exact. As before one defines the derived functors R^pF by $R^pF(A) = H^p(F(A^{\cdot}))$ were $A \to A^{\cdot}$ is a soft resolution of A. This is justified in virtue of the following two propositions, analogous to (1.1) and (1.2).

PROPOSITION 2.1. Each object A of *G-Topab* has a soft resolution.

PROOF: Let $E_{(G)}A = Map(G;\ EA)$. Define a *G*-embedding $A \to E_{(G)}A$ by composing the embedding of A in EA with that of EA in $Map(G;\ EA)$ as the constant functions. Then $A \to E_{(G)}A$ has a local cross-section, so it leads to a short exact sequence $A \to E_G A \to B_G A$ in *G-Topab*. A suitable resolution of A is $A \to E_{(G)}A \to E_{(G)}B_{(G)}A \to E_{(G)}B^2_{(G)}A \to \ldots$.

PROPOSITION 2.2. If $F: G\text{-}Topab \to Ab$ is derivable then $R^pF(A)$ is independent of the soft resolution of A used to define it.

PROOF: To see that the proof of (1.2) applies in this case it suffices to show that the functors $E_{(G)}$ and $B_{(G)}$ are exact. In the case of $E_{(G)}$, this is true because if $A' \to A \to A''$ is a short exact sequence in *G-Topab* then in the induced sequence $EA' \to EA \to EA''$ there is a global cross-section of the fibration $EA \to EA''$ (the group EA being contractible), and so there is an induced global cross-section of the fibration $E_{(G)}A = \text{Map}(G;\ EA) \to E_{(G)}A'' = \text{Map}(G;\ EA'')$. In the case of $B_{(G)}$, a local cross-section of $B_{(G)}A \to B_{(G)}A''$ is given by the composition $p \cdot s_2 \cdot s_1$, where s_1 is a local cross-section of $E_{(G)}A'' \to B_{(G)}A''$, s_2 is a cross-section of $E_{(G)}A \to E_{(G)}A''$, and $p: E_{(G)}A \to B_{(G)}A$ is the projection.

And of course we have

PROPOSITION 2.3. If $F: G\text{-}Topab \to Ab$ is derivable, and $A' \to A \to A''$ is a short exact sequence in *G-Topab*, then there is an infinite exact sequence

$$0 \to F(A') \to F(A) \to F(A'') \to R^1F(A') \to \ldots .$$

EXAMPLE 2.4. The functor $A \mapsto \Gamma^G A$ which associates to an object of *G-Topab* its *G*-invariant subgroup is derivable.

PROOF: Let $A' \to A \to A''$ be a short exact sequence in *G-Topab* with $A' = Map(G; A_0)$. We must show that $\Gamma^G A \to \Gamma^G A''$ is surjective. Let $y \in \Gamma^G A'' \subset A''$, and let $x \in A$ be such that $x \mapsto y$. Then, identifying A' with a subgroup of A, $g \cdot x - x$ is an element $\varphi(g)$ of A' for each $g \in G$. It is easily seen that $\varphi : G \to A'$ is a crossed homomorphism (3): let its adjoint map be $\psi : G \times G \to A_0$. Let ξ be the element of A' defined by $g \mapsto \psi(g^{-1}, 1)$. Then $(g \cdot \xi - \xi) \in A'$ is the map

$$\gamma \mapsto \psi(\gamma^{-1}g, 1) - \psi(\gamma^{-1}, 1).$$

But

$$\psi(\gamma^{-1}g, 1) - \psi(\gamma^{-1}, 1) = \psi(\gamma^{-1}, 1) + \gamma^{-1}\psi(g, 1) - \psi(\gamma^{-1}, 1) = \psi(g, \gamma)$$

because φ is a crossed homomorphism. That is, $g \cdot \xi - \xi = g \cdot x - x$ for all $g \in G$, and so $x - \xi$ is an element of A which maps to y.

§ 3. The cohomology of a discrete group can be calculated from a well-known cochain complex. One can define an analogous *complex of continuous cochains* $C^{\cdot}_G(A)$ in the topological case: one sets $C_G^p(A) = \mathrm{Map}(G \overset{\leftarrow p+1 \rightarrow}{\times \dots \times} G;\ A)$, and defines $d : C_G^p(A) \to C_G^{p+1}(A)$ by the classical formula

$$df(g_0, \dots, g_{p+1}) = \sum_{k+1}^{p+1} (-1)^k f(g_0, \dots, \hat{g}_k, \dots, g_{p+1}).$$

A is embedded in $C_G^0(A) = \mathrm{Map}(G; A)$ as the constant maps. Then $A \to C^{\cdot}_G(A)$ is a resolution of A in *G-Topab*, for there are (non-equivariant) contracting homotopies $h : C_G^{p+1}(A) \to C_G^p(A)$ defined by

$$hf(g_0, \dots, g_p) = f(1, g_0, \dots, g_p).$$

When A is a contractible group the groups $C_G^p(A)$ are soft, so we have

(3) I.e. $\varphi(g_1 g_2) = \varphi(g_1) + g_1 \cdot \varphi(g_2)$ for all $g_1, g_2 \in G$.

PROPOSITION 3.1. If A is contractible then $R^p\Gamma^G(A)$ can be calculated from the complex of continuous cochains.

In general this will not be true, but there is a spectral sequence relating the cohomology of the continuous cochains to the cohomology of G. In fact if $A \to A^{\cdot}$ is a soft resolution of A then from the double complex $\Gamma^G C^{\cdot}_G(A^{\cdot})$ we obtain

PROPOSITION 3.2. There is a spectral sequence with $E_1^{pq} = H^q(G \times \overset{\leftarrow p \rightarrow}{\ldots} \times G;\ A)$ converging to $R^*\Gamma^G(A)$.

PROOF. For each q the complex $\Gamma^G A^q \to \Gamma^G C^{\cdot}_G(A^q)$ is acyclic, so the total cohomology of $\Gamma^G C^{\cdot}_G(A^{\cdot})$ is $R^*\Gamma^G(A)$. On the other hand for any p the complex $\Gamma^G C_G^p(A^{\cdot})$ can be identified with Map $(G \times \overset{\leftarrow p \rightarrow}{\ldots} \times G;\ A^{\cdot})$, and we have seen in (1.4) that the cohomology of this is $H^p(G \times \overset{\leftarrow p \rightarrow}{\ldots} \times G;\ A)$.

REMARKS.

i) Notice that $E_1^{p0} = C_G^p(A)$, so that the edgehomomorphism $E_2^{p0} \to R^p\Gamma^G(A)$ is a natural transformation from the cohomology of the continuous cochains to the cohomology of G.

ii) If, for example, A is discrete and $H_*(G)$ is torsion-free then

$$H^*(G \times \ldots \times G;\ A) = \mathrm{Hom}\,(H_*(G) \otimes \ldots \otimes H_*(G);\ A)$$

and one has the spectral sequence of Eilenberg-Moore-etc.

$$\mathrm{Ext}_{H_*(G)}^p(\mathbf{Z};\ A) \Rightarrow R^*\Gamma^G(A).$$

To an object A of *G-Topab* one can associate a sheaf of abelian groups σA on the space BG by defining $\sigma A(U) = \mathrm{Map}^G(p^{-1}U;\ A)$, where U is an open set of BG and $p: EG \to BG$ is the projection. This gives a functor $\sigma: G\text{-}Topab \to Sh(BG)$, where $Sh(BG)$ is the abelian category of sheaves on BG. It is easy to see that σ takes exact sequences to exact sequences, and soft objects to soft sheaves. So from the natural transformation $\Gamma^G A \to H^0(BG;\ \sigma A)$ one derives natural transformations $R^p\Gamma^G A \to H^p(BG;\ \sigma A)$ for all $p \geqslant 0$. One has a transformation of the spectral sequence of (3.2) into that of the double complex

$$E_0^{pq} = H^0(BG;\ \sigma\, Map\,(G^{p+1};\ A^q)).$$

But

$$H^0(BG;\ \sigma\,\mathrm{Map}\,(G^{p+1};\ A^q)) = \mathrm{Map}^G\,(EG;\ Map\,(G^{p+1};\ A^q)) \cong$$
$$\cong \mathrm{Map}\,(EG \times G^p;\ A^q),$$

so $E_1^{pq} = H^q(EG \times G^p;\ A)$ by (2.4). If A is discrete this spectral sequence coincides at the E_1-level with that of (3.2), because EG is contractible; and one deduces

PROPOSITION 3.3. If A is discrete then $R^p\Gamma^G(A) \cong H^p(BG;\ \sigma A)$.

§ 4. To conclude, I shall show that the cohomology groups in dimensions 1 and 2 have their usual interpretations. Let Hom$(G;\ A)$ denote the abelian group of crossed homomorphisms from G to A modulo principal crossed homomorphisms (i.e. ones of the form $g \mapsto g \cdot a - a$ for some $a \in A$). And let Ext$(G;\ A)$ denote the set of isomorphism-classes of extensions $A \to E \to G$ which have a local cross-section. Ext$(G;\ A)$ is a contravariant functor in G, because $G' \to G$ induces $E \mapsto E \times_G G'$; and a covariant functor in A, because $A \to A'$ induces $E \mapsto (E \times A')/A$. The composition-law

$$\mathrm{Ext}\,(G;\ A) \times \mathrm{Ext}\,(G;\ A) \to \mathrm{Ext}\,(G;\ A \times A)$$

given by $(E_1,\ E_2) \mapsto E_1 \times_G E_2$, together with $A \times A \to A$, makes Ext$(G;\ A)$ into an abelian group.

PROPOSITION 4.1. If $A' \to A \to A''$ is a short exact sequence in *G-Topab* then there is an exact sequence

$$0 \to \Gamma^G A' \to \Gamma^G A \to \Gamma^G A'' \to \mathrm{Hom}\,(G;\ A') \to \mathrm{Hom}\,(G;\ A) \to$$
$$\to \mathrm{Hom}\,(G;\ A'') \to \mathrm{Ext}\,(G;\ A') \to \mathrm{Ext}\,(G;\ A) \to \mathrm{Ext}\,(G;\ A'').$$

PROOF. The first d assigns to $y \in \Gamma^G A''$ the crossed homomorphism $g \mapsto g \cdot x - x$, where $x \in A$ has image y in A''. The second d assigns to a crossed homomorphism $G \mapsto A''$ the extension $G \times_{A''} A$. To verify exactness is trivial.

PROPOSITION 4.3. If A is soft then Hom$(G;\ A) =$ *Ext*$(G;\ A) = 0$.

PROOF. Let $A = Map\,(G;\ A_0)$. Given a crossed homomorphism $f : G \to A$, let its adjoint be $\tilde{f} : G \times G \to A_0$. Then a trivial calculation (done in the proof of (2.4)) shows that $f(g)=g\cdot\varphi-\varphi$, where $\varphi(g)=$ $=f(g^{-1}, 1)$. Thus f is principal and Hom $(G; A)=0$.

Let $A \to E \to G$ be an extension. Because A is contractible there is a continuous cross-section $\varphi : G \to E$. Define $\alpha : G \times G \to A$ so that

$$\varphi(g_1, g_2)=\alpha(g_1, g_2)\varphi(g_1)\varphi(g_2),$$

and let its adjoint be $\tilde{\alpha} : G \times G \times G \to A$. Then if $\beta : G \to A$ has adjoint $\tilde{\beta} : G \times G \to A$, where

$$\tilde{\beta}(g_1, g_2)=g_2\cdot\tilde{\alpha}(g_2^{-1}, g_1, 1),$$

a calculation shows that $g \mapsto \beta(g)\varphi(g)$ is a splitting of the extension E. Hence Ext $(G; A)=0$.

PROPOSITION 4.3. $R^1\Gamma^G(A) \cong \mathrm{Hom}(G; A)$, and $R^2\Gamma^G(A) \cong \mathrm{Ext}(G; A)$.

PROOF. Because of the exact sequence $A \to E_{(G)}A \to B_{(G)}A$, both $R^1\Gamma^G(A)$ and Hom $(G;$ A) can be identified with the cokernel of

$$\Gamma^G(E_{(G)}A) \to \Gamma^G(B_{(G)}A),$$

so they are isomorphic. And similarly

$$R^2\Gamma^G(A) \cong R^1\Gamma^G(B_{(G)}A),$$

while

$$\mathrm{Ext}\,(G;\ A) \cong \mathrm{Hom}\,(G;\ B_{(G)}A).$$

Appendix (A).

In [6] I explained how one can associate naturally to a topological space A a contractible space EA obtained from the semi-simplicial space $\{A_n\}$, where $A_n=A \times \overset{\leftarrow n+1 \rightarrow}{\dots} \times A$, by the usual realization process; i.e. EA is obtained from the topological sum $\coprod_n A_n \times \Delta^n$, where Δ^n is the standard n-simplex, by making identifications along the boundaries of the simplexes.

Another way of describing EA is as follows: it is the space of step-functions on the unit interval I with values in A, where by step-function one means a function constant on each of the half-open intervals

$(t_i, t_{i+1}]$ for some partition $0=t_0<t_1< \dots <t_n=1$ of I. But from this point of view the topology seems rather obscure.

If A and B are two spaces then $E(A \times B) \cong EA \times EB$ in the category of k-spaces, so that EA is a group in the category if A is one. When EA is thought of as a function-space the composition-law is the obvious one, and EA admits a continuous monomorphism on to a dense subgroup of $L^1(I:A)$. The group A is embedded in EA as a closed subgroup (the constant functions) which is normal if and only if A is abelian. One defines $BA = EA/A$. The following proposition replaces the vague remark on the same subject in [6].

PROPOSITION (A.1). If A is a locally contractible group (not necessarily abelian) then it has a local cross-stection in EA, so that the fibration $A \to EA \to BA$ is locally trivial. (Note: « Locally contractible » means that for each neighbourhood U of each point x there is a neighbourhood V of x contained in U which is contractible in U).

PROOF. BA is the realization of the semi-simplicial space $\{B_n\}$, where $B_n = A_n/A = \overleftrightarrow{A \times \overset{n}{\dots} \times A}$. Let B^nA be the n-skeleton [6] of BA, which is obtained by attaching $B_n \times \Delta^n$ to $B^{n-1}A$ by a certain map

$$(B_n \times \dot{\Delta}^n) \cup (B_n^d \times \Delta^n) \to B^{n-1}A,$$

where B_n^d is the degenerate part of B_n. Suppose that a cross-section of $E^{n-1}A \to B^{n-1}A$ has been found in a neighbourhood of the base-point in $B^{n-1}A$. When the bundle $EA \to BA$ is pulled back to $B_n \times \Delta^n$ it becomes trivial, so the problem of extending the partial section reduces to that of extending a map $V \to A$, where V is an open set of $(B_n \times \dot{\Delta}^n) \cup (B_n^d \times \Delta^n)$, to a neighbourhood of V in $B_n \times \Delta^n$. That can be done because $(B_n \times \dot{\Delta}^n) \cup (B_n^d \times \Delta^n)$ is a neighbourhood deformation retract in $B_n \times \Delta^n$; which is true in turn because

$$B_n^d = \{(a_i, \dots, a_n) \in A \times \dots \times A : a_i = 1 \text{ for some } i\}$$

is a neighbourhood deformation retract in B_n if A is locally contractible.

PROPOSITION (A.2). If A is a locally contractible group then EA and BA are locally contractible.

PROOF. EA is locally contractible because any contractible topological group is locally contractible. The local contractibility of BA follows from that of EA in view of the local cross-section found in (A.1).

Proposition (A.3). If $A' \to A \to A''$ is a short exact sequence in *Topab* then the induced sequences $EA' \to EA \to EA''$ and $BA' \to BA \to$ $\to BA''$ are exact.

Proof. The algebraic exactness of the sequences is trivial. The local cross-section of $A \to A''$ induces a local cross-section of $EA \to EA''$, because E is a functor on topological spaces. A local cross-section of $BA \to BA''$ can be obtained from local sections of $EA'' \to BA''$ and $EA \to EA''$.

Appendix (B).

I shall prove the following theorem, which is true either in the category of topological spaces or in the category of k-spaces.

Theorem (B.1). If A' is a contractible closed subgroup of a topological group A which has a local cross-section then it has a global cross-section, i.e. $A = A' \times (A/A')$ as topological space. (Notice that A' and A are not assumed abelian).

The proof depends on the following lemma inspired by [4].

Proposition (B.2). If X is a uniform space, and $U \subset X \times X$ is an entourage of the diagonal, then the covering $\{U_x\}_{x \in X}$ of X is numerable [1], where $U_x = \{y \in x : (x, y) \in U\}$.

Example. If G is a topological group and V is a neighbourhood of the identity in G then the covering $\{gV\}_{g \in G}$ of G is numerable.

Proof of (B.2). Mather ([4] Thm. 1) has shown that is enough to find functions $\varphi_x : X \to [0, 1]$ for each $x \in X$ such that $\sum_{x \in X} \varphi_x(y) = 1$ for all $y \in X$ and $\varphi_x(y) = 0$ when $y \notin U_x$. Choose an écart f on X such that $y \in U_x \Leftrightarrow f(x, y) < 1$, and let $\theta_x : X \to [0, 1]$ be defined by

$$\theta_x(y) = \sup (1 - f(x, y), 0).$$

Well-order the points of X and define

$$\varphi_x(y) = \theta_x(y) - \sup_{z < x} \theta_z(y).$$

Then φ_x is continuous because the functions $\{\theta_z\}$ are uniformly equicontinuous and hence have a continuous supremum. Clearly $\Sigma \varphi_x = 1$, and $\varphi_x(y) = 0$ if $y \notin U_x$.

Proof of (B.1). Because of (B.2) the fibration $A' \to A \to A'' = A/A'$ is trivial over each of the sets of a numerable covering of A'', so by a well-known argument of Milnor (cf. [5], 3.6) one can suppose it is trivial over the sets $\{U_k\}_{k \in \mathbf{N}}$ of a locally finite countable covering. Let

$$W_k = U_0 \cup U_1 \cup \dots \cup U_{k-1}.$$

Suppose one has found a cross-section $w_k : W_k \to A$, and has a cross-section $u_k : U_k \to A$. Then one can define $w_{k+1} : W_{k+1} \to A$ by

$$\begin{aligned} w_{k+1} &= w_k(x) \text{ for } x \in W_k - U_k \\ &= u_k(x) \text{ for } x \in U_k - W_k \\ &= w_k(x) \cdot c(u_k(x) \cdot w_k(x)^{-1}, \theta(x)), \text{ for } x \in W_k \cap U_k \end{aligned}$$

where $c : A' \times [0, 1] \to A'$ is a contraction and $\theta : X \to [0, 1]$ is a function equal to 1 on $W_k - U_k$. Finally, define the desired cross-section $w : A'' \to A$ by $w(x) = w_k(x)$ for large k: that is permissible because $w_k(x) = w_{k+1}(x)$ if $x \notin U_k$, and the covering $\{U_k\}$ is locally finite.

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