Higher Poisson brackets

Def: $n : \mathbb{N}$. A pre n-plectic mfld (X, ω) is a mfld w/ $\omega : \Omega^{n+1}_{cl}(X)$

$\Rightarrow$ it is <u>n-pletic</u>

s.t. ω gives an <u>injection</u>

Def: Let (X, ω) - Pre n-pletic

$v : \mathcal{X}(X) \ \& \ h : \Omega^{n-1}(X)$

s.t. $\iota_v \omega + dh = 0$

Then v is a Hamiltonian v.f.

h is a <u>Ham. fun for v</u>

$$\Omega^{n-1}_{Ham}(X) = \{(v, h) : \mathcal{X}(X) \oplus \Omega^{n-1}(X) \mid \iota_v \omega + dh = 0\}$$

the higher Poisson bracket L_∞-alg of local observ.)

Def: Let (X, ω) be a pre-n-pletic

we denote by $\mathfrak{Poisson}(X, \omega)$

the L_∞-algebra

$$\to \Omega^0(X) \to \Omega^1(X) \to \dots \to \Omega^{n-1}_{Ham}(X)$$

$\Rightarrow$ Only non-vanishing brackets on $(v, h) : \Omega^{n-1}_{Ham}(X, \omega)$

$$[(v, h), (\tilde{v}, \tilde{h})] =$$

$$= ([v, \tilde{v}], \iota_{v_1} \iota_{v_2} \omega)$$

Ex) For $n=1$, non-deg ω this is the traditional Poisson bracket Lie Alg on $C^\infty(X)$ of X

$$\Omega^0_{Ham}(X) \longrightarrow C^\infty(X)$$

$$(v, h) \longmapsto h$$

$$(X_h, h) \longleftarrow h$$

$$([X_{h_1}, X_{h_2}], \{h_1, h_2\}) \longleftarrow \{h_1, h_2\}$$

$$\overset{?}{=} \iota_{X_{h_1}} \iota_{X_{h_2}} \omega$$

Def: prequantization

$$\begin{tikzcd} & \mathbf{B}^n U(1)_{conn} \arrow[d, "curv"] \\ X \arrow[ur, dashed] \arrow[r, "\omega"'] & \Omega^{n+1}_{cl} \end{tikzcd}$$

Prop: Let ∇ be a prequantization of (X,ω). Let <u>quantomorph</u>(∇) be the L_∞-alg of inf. quantomorph. of ∇

Thm (Higher Kostant-Souriau ext.)

for (X,ω): pre n-plectic mfld

$$\begin{tikzcd}
\mathbf{H}(X,\flat\mathbf{B}^{n-1}\mathbb{R}) \arrow[r] \arrow[d] & \underline{\mathrm{Poisson}}(X,\omega) \arrow[r] \arrow[d] & 0 \arrow[d] \\
0 \arrow[r] & \mathcal{X}_{\mathrm{Ham}}(X,\omega) \arrow[r] & \mathbf{BH}(X,\flat\mathbf{B}^{n-1}\mathbb{R})
\end{tikzcd}$$

This yields a higher Heisenberg <u>cocycle</u>.

Ex: for $\mathfrak{g}$ an L_∞-algebra a Hamiltonian action of $\mathfrak{g}$ on X is an L_∞-morphism

$$\rho : \mathfrak{g} \longrightarrow \mathcal{X}_{\mathrm{Ham}}(X)$$

$$
\begin{tikzcd}
\underline{\mathfrak{heis}_\rho(\mathfrak{g})} \arrow[r] \arrow[d] \arrow[dr, phantom, "\lrcorner_h", very near start] & \underline{\mathrm{Poisson}}(X,\omega) \arrow[d] \\
\mathfrak{g} \arrow[r] & \mathfrak{X}_{\mathrm{Ham}}(X)
\end{tikzcd}
$$