

January 28: Simplicial sets and model categories I (Stephen Peña)

- The definition of simplicial sets as presheaves on the simplex category.
- Examples: Δ^n , Λ_k^n , $\partial\Delta^n$. Nerves of categories and groupoids.
- Simplicial maps.
- Kan complexes.
- Limits and colimits of simplicial sets. Example: products.
- Simplicial homotopies.
- Homotopy groups of Kan complexes. Simplicial homotopy equivalences.
- Ex^∞ and simplicial weak equivalences (if time permits).

References

- Pavlov: Topology. <https://dmitripavlov.org/notes/2018f-5324.pdf>
- Pavlov: Homological algebra I. <https://dmitripavlov.org/notes/2019f-6321-handwritten.pdf>

February 4: Simplicial sets and model categories II (Gregory Taroyan)

- Definition of relative categories, relative functors, and the hammock localization.
- Examples: simplicial sets and simplicial weak equivalences; chain complexes and quasi-isomorphisms.
- Model categories: definition.
- Examples of model categories: the Kan–Quillen model structure on simplicial sets; very briefly mention the projective model structure on chain complexes (will be treated separately on February 18).
- Enriched and monoidal model categories: definition, examples: simplicial sets, (very briefly) chain complexes.
- Derived functors of Quillen adjoint functors: definition, examples: homotopy products and pullbacks (illustrate using simplicial sets), quotients and homotopy cofibers.
- Derived mapping simplicial sets compute the hammock localization in a simplicial model category.

References

- Hovey: Model categories.
- Joyal: Model categories. <https://ncatlab.org/joyalscatlab/published/Model+categories>
- Pavlov: Topology. <https://dmitripavlov.org/notes/2018f-5324.pdf>
- Pavlov: Homological algebra I. <https://dmitripavlov.org/notes/2019f-6321-handwritten.pdf>

February 11: Simplicial sets and model categories III. (James Francese)

- Spillover from the first two talks.

References

- Hovey: Model categories.
- Joyal: Model categories. <https://ncatlab.org/joyalscatlab/published/Model+categories>
- Pavlov: Topology. <https://dmitripavlov.org/notes/2018f-5324.pdf>
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February 18: The Apocalypse (Snow storm)

February 25: Homological algebra and model categories of chain complexes. (Ramiro Ramirez)

- Definition of chain complexes, chain maps, chain homotopy equivalences, quasi-isomorphisms.
- Definition of the projective and injective model structures on chain complexes.
- Explicit description of cofibrations, fibrations, acyclic cofibrations, acyclic fibrations.
- Explicit description of cofibrant/fibrant replacements in the projective/injective model structure as projective/injective resolutions.
- Reference: Hovey: 2.3.

- Definition of left/right Quillen functors and Quillen bifunctors.
- Example: the tensor-hom Quillen adjunction for chain complexes.
- Reference: Hovey: 4.2.

- A very brief crash-course on derived functors of left/right Quillen functors (e.g., something like in §41 and §48 of [HA1]), including a definition of the homotopy category of a model category, and the fact that left/right Quillen functors induce functors between homotopy categories.
- Reference: Hovey: 1.2, 1.3.
- The derived tensor product and derived hom of chain complexes.
- Tor and Ext.
- Example: compute $\mathrm{Tor}_n^{\mathbf{Z}}(\mathbf{Z}/m, A)$, $\mathrm{Ext}_{\mathbf{Z}}^n(\mathbf{Z}/m, A)$, $\mathrm{Ext}_{\mathbf{Z}}^n(A, \mathbf{Z}/m)$ for all $n \in \mathbf{Z}$ and $m \in \mathbf{Z}$, where A is an abelian group.
- Reference: Weibel: An Introduction to Homological Algebra, Chapter 3.

References

- Hovey: Model categories.
- Dwyer and Spalinski: Homotopy theories and model categories (especially §7 and §9)
- Weibel: An introduction to homological algebra.
- Schreiber: Introduction to Homological Algebra.
<https://ncatlab.org/schreiber/show/Introduction+to+Homological+Algebra>
- [HA1] Pavlov: Homological algebra I. <https://dmitripavlov.org/notes/2019f-6321-handwritten.pdf>

March 4: Differential graded algebras (James Francese)

- The projective model structure on commutative DGAs.

References

- Bousfield and Gugenheim: On PL de Rham theory and rational homotopy type.
- Pavlov and Scholbach: Admissibility and rectification of colored symmetric operads.

March 11: Rational homotopy theory (James Francese)

References

- Félix, Halperin, Thomas: Rational Homotopy Theory; Rational Homotopy Theory II.
- Sullivan: Infinitesimal computations in topology.
- Quillen: Rational homotopy theory.
- Bousfield, Gugenheim: On PL de Rham theory and rational homotopy type.

March 18: Differential graded C^∞ -rings (Dmitri Pavlov)

- Definition of C^∞ -rings.
- Examples: smooth functions on smooth manifolds, Weil algebras, formal power series, germs.
- Constructions: ideal and quotients, tensor products, colimits and limits, derivations, Kähler differentials.
- Definition of differential graded C^∞ -rings. Model structure. Constructions.
- Examples of derived intersections: intersect $x^p = y^q$ with $x^a = y^b$, intersect $y = 0$ with $y = \exp(-x^{-2})$.

References

- Moerdijk and Reyes: Models for smooth infinitesimal analysis.
- Carchedi and Roytenberg: On theories of superalgebras of differentiable functions.
- Carchedi and Roytenberg: Homological algebra for superalgebras of differentiable functions.
- Pavlov: Homological algebra II. <https://dmitripavlov.org/notes/2020s-6322-handwritten.pdf>

March 25: Differential graded C^∞ -rings: model structures and examples of computations (Dmitri Pavlov)

References

April 1: Differential graded C^∞ -rings: examples of computations (Dmitri Pavlov)

References

April 8: Derived differentiable stacks (Dmitri Pavlov)

- The relative category of simplicial presheaves on a relative site.
- Model structures on simplicial presheaves.
- Explicit description of fibrant objects (derived differentiable stacks).
- Example: the de Rham complex of a derived differentiable stack.

References

General references on sheaves of sets on a site:

- Mac Lane and Moerdijk: Sheaves in geometry and logic.
- Borceux: Handbook of categorical algebra 3.

General references on simplicial presheaves:

- Pavlov: Homological algebra II. <https://dmitripavlov.org/notes/2020s-6322-handwritten.pdf>
- Jardine: Local homotopy theory.

General references on derived stacks:

- Toën and Vezzosi: Homotopical algebraic geometry I: topos theory.
- Toën and Vezzosi: Homotopical algebraic geometry II: geometric stacks and applications.
- Toën: Higher and derived stacks: a global overview.

April 15: K-theory of derived differentiable stacks (Daniel Grady)

April 22: The Batalin-Vilkovisky formalism. The BV-BRST formalism. (Stephen Peña)

References

- Schreiber: Geometry of physics.
- Pavlov: Homological algebra II. <https://dmitripavlov.org/notes/2020s-6322-handwritten.pdf>

April 29: The Batalin-Vilkovisky formalism. The BV-BRST formalism. (James Francese)

References

- Schreiber: Geometry of physics.
- Pavlov: Homological algebra II. <https://dmitripavlov.org/notes/2020s-6322-handwritten.pdf>