Mathematics 5317/5399 (Fall 2020 / Spring 2021): Introduction to Modern Algebra I/II

Instructor: Dmitri Pavlov, Assistant Professor
CRN: 42450
Lectures: TuTh 9:30–11 a.m., MATH 109
Website: https://dmitripavlov.org/#teaching
Office hours: by appointment
Email: dmitri.pavlov@ttu.edu
Textbook: Paolo Aluffi. Algebra: Chapter 0. AMS, 2009.
Supplementary books:
- F. William Lawvere, Robert Rosebrugh. Sets for Mathematics. Cambridge University Press, 2003.
- Werner Greub. Multilinear Algebra (2nd Edition). Springer, 1978.
- Nicolas Bourbaki. Algebra I. Chapters 1–3. Springer, 1989.

1 Course outline

Graduate-level introduction to the theory of groups and rings.

This class covers those areas of linear algebra, multilinear algebra, and elementary algebra of groups, rings, and modules that are crucial for other graduate classes, such as Functional Analysis (5340/5341), Topology (5324/5325), Geometry (5328/5329; to be created), and Algebra (5326/5327).

2 Expected learning outcomes

Upon the completion of this course students will be able to apply the following concepts in their research:
- definitions of groups, rings, modules, vector spaces, algebras (including matrices); homomorphisms;
- free groups, abelian groups, rings, modules;
- generators and relations for groups, rings, modules;
- quotients of groups, rings, modules; their universal properties;
- subgroups and normal subgroups, subalgebras and ideals, submodules;
- first, second and third isomorphism theorem for groups, rings, modules; exact sequences;
- group actions; stabilizers, normalizers, and centralizers; semidirect products;
- Euclidean domains, unique factorization domains, principal ideal domains;
- products and coproducts of groups, rings, modules; their universal properties; limits and colimits;
- tensor products of modules, hom-tensor adjunction, tensor algebra, symmetric algebra;
- polynomials, rational functions, formal power series, localization of algebras and modules;
- Noetherian rings; localizations of rings and modules; radical ideals; Stone duality; Zariski spectrum;
- exterior algebra and its geometric meaning, Hodge star; determinants; Pfaffian;
- derivations; Kähler differentials; de Rham complex;
- bilinear, hermitian, and quadratic forms; spectral theorems; law of inertia, signature; singular value decomposition; Clifford algebras and their periodicity;
- modules over principal ideal domains; eigenspaces, Jordan normal form and solution of constant-coefficient ODEs; Smith normal form; classification of finitely generated abelian groups.

3 Assessment of learning outcomes

Homework will be assigned throughout the semester. Incorrect homework solutions can be resubmitted with corrections. All homework must be submitted on or before the last day of classes. Each correctly solved homework problem is worth 1 point, unless otherwise indicated.

Additionally, in order to develop their mathematical writing skills, students are required to take notes for at least 3 classes of their choice and typeset them in TEX. The resulting text will be assessed for correctness and corrections should be made as necessary. Fully corrected notes for a single class are worth 10 points.

4 Criteria for grade determination

The final grade is determined by the total number of points according to the following rubric:

$$[0,60) \mapsto C \quad [60,80) \mapsto B, \quad [80,100) \mapsto A, \quad [100,\infty) \mapsto A+.$$

5 Schedule

There will be a total of 28 class meetings on the following days:
August 25:
August 27:
September 1:
September 3:
September 8:
September 10:
September 15:
September 17:
September 22:
September 24:
September 29:
October 1:
October 6:
October 8:
October 13:
October 15:
October 20:
October 22:
October 27:
October 29:
November 3:
November 5:
November 10:
November 12:
November 17:
November 19:
November 24:
December 1:

6 Operating policy 34.19: Student absence for observance of religious holy day

1. "Religious holy day" means a holy day observed by a religion whose places of worship are exempt from property taxation under Texas Tax Code §11.20.

2. A student who intends to observe a religious holy day should make that intention known in writing to the instructor prior to the absence. A student who is absent from classes for the observance of a religious holy day shall be allowed to take an examination or complete an assignment scheduled for that day within a reasonable time after the absence.

3. A student who is excused under section 2 may not be penalized for the absence; however, the instructor may respond appropriately if the student fails to complete the assignment satisfactorily.

7 Operating policy 34.22, §2a: Reasonable accommodation for students with disabilities

Any student who, because of a disability, may require special arrangements in order to meet the course requirements should contact the instructor as soon as possible to make any necessary arrangements. Students should present appropriate verification from Student Disability Services during the instructor's office hours. Please note: instructors are not allowed to provide classroom accommodations to a student until appropriate verification from Student Disability Services has been provided. For additional information, please contact Student Disability Services in West Hall or call 806.742.2405.

8   Operating policy 34.12, §4: Texas Tech University Statement of Academic Integrity

Academic integrity is taking responsibility for one's own class and/or course work, being individually accountable, and demonstrating intellectual honesty and ethical behavior. Academic integrity is a personal choice to abide by the standards of intellectual honesty and responsibility. Because education is a shared effort to achieve learning through the exchange of ideas, students, faculty, and staff have the collective responsibility to build mutual trust and respect. Ethical behavior and independent thought are essential for the highest level of academic achievement, which then must be measured. Academic achievement includes scholarship, teaching, and learning, all of which are shared endeavors. Grades are a device used to quantify the successful accumulation of knowledge through learning. Adhering to the standards of academic integrity ensures grades are earned honestly. Academic integrity is the foundation upon which students, faculty, and staff build their educational and professional careers. (Texas Tech University Quality Enhancement Plan, Academic Integrity Task Force, 2010.)