

Math 2360-121

Review Session 3

Midterm 2, Problem 1

Find the coordinates of $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

with respect to the basis

$$\begin{pmatrix} 7 \\ 9 \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \end{pmatrix}.$$

Solution The vector with

coordinates c_1, c_2

with respect to the

basis $\begin{pmatrix} 7 \\ 9 \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ is

$$c_1 \begin{pmatrix} 7 \\ 9 \end{pmatrix} + c_2 \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 7c_1 + 3c_2 \\ 9c_1 - c_2 \end{pmatrix}.$$

We want to find c_1, c_2

such that

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 7c_1 + 3c_2 \\ 9c_1 - c_2 \end{pmatrix},$$

$$\text{i.e., } \begin{cases} 7c_1 + 3c_2 = 1 \\ 9c_1 - c_2 = 2 \end{cases}$$

$$34c_1 = 7 \quad c_1 = \frac{7}{34}$$

$$c_2 = 9c_1 - 2 = \frac{63 - 2 \cdot 34}{34} = \frac{-5}{34}.$$

Answer: $\frac{7}{34}, \frac{-5}{34}$.