Mathematics 6333 (Fall 2019): Introduction to Lie Groups and Their Representations

Instructor: Dmitri Pavlov, Assistant Professor
Lectures: TuTh 11–12:30, MA 113
Credit hours: 3
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1 Course Outline

This course will teach the basics of Lie theory, understood in the broadest possible sense, including classical Lie theory, higher Lie theory, its applications in physics, including Chern–Simons theory, and other topics.

A classical array of topics will be covered, including the following: smooth manifolds, infinitesimals, differential forms and differential operators, Lie groups and Lie algebras, their representations (including the Peter–Weyl theorem), principal bundles over Lie groups, connections, Chern–Simons theory.

The exposition will be heavily geometric, and connections to other areas of mathematics will be strongly emphasized, including, but not limited to, differential geometry, representation theory, real analysis, complex analysis, number theory, algebraic geometry, partial differential equations, and topology.

2 Catalog Course Description

Lie groups, Lie algebras, exponential map, Lie brackets, representation theory with examples, Peter-Weyl theorem, homogenous and symmetric spaces, applications to ODEs/PDEs arising in physics.

3 Course Specific Expected Learning Outcomes

Upon completion of this course, students will be able to apply Lie theory to their area of research.

4 Assessment of Expected Learning Outcomes

Homework assignments will be given throughout the course. Two midterms and a final may be administered in class or as 4-hour take-home exams around September 26, October 31, and December 3 (tentative and subject to change or elimination).

5 Grading

The final grade depends on the homework, midterms, and final exam, if any of these were administered.

6 Text

The material will be drawn from a variety of sources, individual texts will be indicated as the course progresses.

Some recommended texts include the following:

6.1. *Smooth manifolds*

Michael Spivak: Calculus on manifolds
John M. Lee: Introduction to smooth manifolds, Chapters 1 and 2
Vladimir Arnold: Mathematical methods of classical mechanics, Chapter 7
Jeffrey M. Lee: Manifolds and differential geometry
S. Ramanan: Global Calculus
Amiya Mukherjee: Differential Topology
Raoul Bott, Loring W. Tu: Differential forms in algebraic topology
Jet Nestruev: Smooth manifolds and observables
Werner Greub, Stephen Halperin, Kay Vanstone: Connections, curvature, and cohomology
Frank W. Warner: Foundations of differentiable manifolds and Lie groups

6.2. *Lie groups*

Richard Borcherds, Mark Haiman, Theo Johnson-Freyd, Nicolai Reshetikhin, Vera Serganova: Berkeley lectures on Lie groups and quantum groups

Jean-Pierre Serre: Lie algebras and Lie groups

Terence Tao: Hilbert's fifth problem and related topics

Eckhard Meinrenken: Clifford algebras and Lie theory

Anthony W. Knapp: Lie groups, Lie algebras, and cohomology

7   Schedule

There will be 28 class meetings on the following days:

August 27:

August 29:

September 3:

September 5:

September 10:

September 12:

September 17:

September 19:

September 24:

September 26:

October 1:

October 3:

October 8:

October 10:

October 15:

October 17:

October 22:

October 24:

October 29:

October 31:

November 5:

November 7:

November 12:

November 14:

November 19:

November 21:

November 26:

December 3:

8   Operating Policy 34.19: Student absence for observance of religious holy day

1. "Religious holy day" means a holy day observed by a religion whose places of worship are exempt from property taxation under Texas Tax Code §11.20.

2. A student who intends to observe a religious holy day should make that intention known in writing to the instructor prior to the absence. A student who is absent from classes for the observance of a religious holy day shall be allowed to take an examination or complete an assignment scheduled for that day within a reasonable time after the absence.

3. A student who is excused under section 2 may not be penalized for the absence; however, the instructor may respond appropriately if the student fails to complete the assignment satisfactorily.

9 Operating Policy 34.22(2b): Reasonable accommodation for students with disabilities

Any student who, because of a disability, may require special arrangements in order to meet the course requirements should contact the instructor as soon as possible to make any necessary arrangements. Students should present appropriate verification from Student Disability Services during the instructor's office hours. Please note: instructors are not allowed to provide classroom accommodations to a student until appropriate verification from Student Disability Services has been provided. For additional information, please contact Student Disability Services in West Hall or call 806–742–2405.

10 Operating Policy 34.12(5): Academic dishonesty definitions

Students must understand the principles of academic integrity, and abide by them in all class and/or course work at the University. Academic Misconduct violations are outlined Part I, section B.1 of the Code of Student Conduct. If there are questions of interpretation of academic integrity policies or about what might constitute an academic integrity violation, students are responsible for seeking guidance from the faculty member teaching the course in question.

Academic misconduct includes cheating, plagiarism, collusion, falsifying academic records, misrepresenting facts, violations of published professional ethics/standards, and any act or attempted act designed to give unfair academic advantage to oneself or another student. Additional information about academic misconduct is available in the Texas Tech University Handbook in Part II, section B of the Community Policies section in the Student Handbook at http://www.depts.ttu.edu/dos/handbook/.