HOMEWORK [LIE GROUPS]

EXERCISE #1 Prove that the torus  is a smooth manifold

EXERCISE #2a Show that a smooth manifold (defined as in Predefinition 1) is a smooth manifold in the sense of Predefinition 2. ↑

EXERCISE #2b Show that the "forgetful functor" between Predefinition 1 smooth manifolds and Predefinition 2 smooth manifolds induces a fully faithful functor of categories

EXERCISE #3 Show that $SL(\mathbb{R}^n)$ [where $SL(\mathbb{R}^n) = \{g \in GL(\mathbb{R}^n) \mid \det g = 1\}$] is a Lie group

EXERCISE #4a Prove that $\mathbb{RP}^n$ [where $\mathbb{RP}^n = S^n/(x \sim -x)$] is a smooth manifold.

EXERCISE #4b Show that the quotient map $S^n \longrightarrow \mathbb{RP}^n$ is smooth (C^∞).

HOMEWORK [LIE GROUPS]

EXERCISE #5 Define a functor from Predefinition 2 to Predefinition 3 that is an equivalence of categories.

EXERCISE #6 Complete the construction of the functor

$$\mathrm{Spec} : \mathrm{SAlg}_{\mathbb{R}} \longrightarrow \mathrm{Man}$$

(the adjoint of the global sections functor Γ.)

EXERCISE #7

Using the definition of the "derivation," compute tangent vectors to $M \subset \mathbb{R}^n$ (an embedded manifold).

EXERCISE #8 Formalize the Möbius bundle as a vector bundle over $M \in \mathrm{Man}$.

EXERCISE #9 Show that the functor defined from vector bundles in Predefinition 2 to vector bundles in Predefinition 3 is an equivalence of categories

$$\mathrm{Vect}_M ② \longrightarrow \mathrm{Vect}_M ③$$

$$ts(V) \xrightarrow{p} M \longmapsto V(U) := \{ s : U \xrightarrow{C^\infty} ts(V) \mid ps = id_U \}$$