HOMEWORK

[HOMOLOGICAL ALGEBRA]

EXERCISE #1

Let $X =$

$Y_1 =$ and $Y_2 =$

Fill in the details (as discussed in class) of why

$\mathrm{Hom}(X, Y_1) \longrightarrow \mathrm{Hom}(X, Y_2)$

is NOT a weak equivalence.

EXERCISE #2

Show that Group (the category of groups...) is <u>NOT</u> cartesian closed.

HOMEWORK

[HOMOLOGICAL ALGEBRA]

EXERCISE #3 Prove that

$$\Delta^n \text{ is Kan} \iff n=0$$

EXERCISE #4 If $\mathcal{C} \in \mathrm{cat}$ (thus $N\mathcal{C} \in \mathrm{sSet}$)

Prove that

$$N\mathcal{C} \text{ is Kan} \iff \mathcal{C} \text{ is a groupoid}$$

EXERCISE #5 Is the map $\mathrm{sd}(\mathrm{sd}\,\Delta^2) \longrightarrow \mathrm{sd}\,\Lambda^2_j$ constructed in the proof of the proposition "$\mathrm{Ex}^\infty X$ is Kan" unique?

EXERCISE #6 Show that the lifting condition in the Simplicial Whitehead Theorem is equivalent to the following two conditions:

(i) the map $\pi_0 f : \pi_0 X \longrightarrow \pi_0 Y$ is an isomorphism of sets

(ii) the map $\pi_n(f, x) : \pi_n(X, x) \longrightarrow \pi_n(Y, f(x))$ is an isomorphism of sets (or groups) for any $n \geq 1$ and any vertex $x \in X_0$

EXERCISE #7 Define X as an infinite chain of vertices:

$$\cdots \bullet \longrightarrow \bullet \longrightarrow \bullet \longrightarrow \bullet \longrightarrow \bullet \longrightarrow \cdots$$
$$-2 \quad -1 \quad 0 \quad 1 \quad 2$$

with $[1]$-simplices connecting vertex n and $n+1$

Consider $f : X \longrightarrow \Delta^0$.

(a) show that f is a simplicial weak equivalence

(b) Show that

$$\prod_{n \geq 0} f : \prod_{n \geq 0} X \longrightarrow \Delta^0$$

is NOT a simplicial weak equivalence.
[Hint: compute π_0]

EXERCISE #8 Show that the functor

$$sSet^I \xrightarrow{\prod} sSet$$

preserves simplicial homotopy equivalences.

HOMEWORK [Homological Algebra]
(RHom-work)

EXERCISE #9 Let I be the discrete category, $\mathcal{C} = \mathrm{sSet}$ and $W =$ simplicial weak equivalences.

Show that in $(\mathcal{C}, W)^I$,

$$L\operatorname{colim}_I \equiv \operatorname{colim}_I$$

(recall, when I is discrete,

$\operatorname{colim}_I = \coprod_I$; I can be either finite or infinite)

EXERCISE #10 $X \in \mathrm{sSet}$

Given $X \twoheadrightarrow \Delta^1$
(a Kan fibration)
show that $X_0 \simeq X_1$
(where $X_0 = X \times_{\Delta^1} 0$ and $X_1 = X \times_{\Delta^1} 1$)

EXERCISE #11

Show that in Set, monomorphisms coincide with injections