Topology Doctoral Preliminary Examination

August 2020

Work all problems. Give complete proofs and computations for all your answers and examples.

1. Compute the cohomology ring with coefficients in a commutative ring A of the connected sum of $S^1 \times S^1 \times S^1$ with itself.

2. For every $g \geq 0$, compute the Poincaré duality isomorphism with coefficients in a commutative ring A of the orientable surface of genus g.

3. Classify all local systems with typical fiber $\mathbf{Z}$ on the real projective plane and compute the twisted homology with coefficients in these systems.

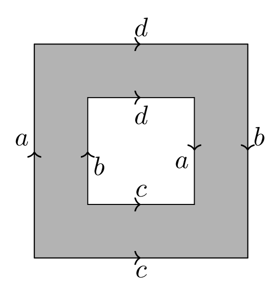
- 1/3 point for classifying local systems, 2/3 point for twisted homology.

4. For every $g \geq 1$ determine whether the nonorientable surface with g crosscaps admits a connected covering that is not a Galois covering and whether the nonorientable surface with g crosscaps admits a connected covering such that the cardinality of its deck transformation group is not equal to the degree of the covering.

- 1/2 point for each part.

5. Classify the following spaces into homotopy equivalence classes (for all $n \geq 2$): $S^n \times S^n$, $S^n \vee S^n \vee S^{2n}$, $(S^n \times S^n) \setminus \{*\}$, $S^n \vee S^n \vee (S^{2n} \setminus \{*\})$. Here $\vee$ denotes the wedge of pointed spaces, i.e., their disjoint union with all basepoints identified, and $\setminus\{*\}$ denotes the removal of a point in the space, different from the basepoint. Within every homotopy equivalence class, further classify the spaces into homeomorphism classes.

6. Consider the following surface given by the area between the two squares, with labeled sides glued when the labels are the same while matching the orientation of edges:



Is this manifold orientable or not? Compute its cohomology ring and Poincaré duality isomorphism. What surface in the classification of surfaces (recall its statement) does this surface correspond to?