Mathematics 6325 (Spring 2018): Category Theory

Instructor: Dmitri Pavlov, Assistant Professor
Lectures: MWF 9–9:50 a.m., MA 10
Office hours: MWF 3–4 p.m., MA 19D, or by appointment
Website: https://dmitripavlov.org/
Email: dmitri.pavlov@ttu.edu

1 Course description

The following basic topics are guaranteed to be covered:

- categories, functors, natural transformations, adjunctions;
- limits and colimits;
- presheaves, representable functors, the Yoneda lemma.

Additional topics will be selected based on the interests of students in the class.

2 Student learning outcomes

Upon the completion of this course students will be able to solve problems in their area of specialization and other branches of mathematics using category theory.

3 Prerequisites

Some knowledge of abstract mathematics will be beneficial. The specific choice of topics is not important. Some examples include:

- algebra (vector spaces, rings, groups, modules);
- general topology, measure theory, functional analysis (topological spaces, measure spaces, topological vector spaces, Banach spaces);
- geometry (manifolds and varieties, vector bundles);
- topology (homotopy groups, homology and cohomology);
- analysis (differential operators, pseudodifferential operators, kernels);
- computer science (complexity theory, algorithms);
- numerical methods (finite elements).

4 Assessment of learning outcomes

The learning outcomes will be assessed based on a combination of the following factors:

- performance on homework;
- final exam;
- a written report on a project in category theory and its presentation in class.

Only a subset of the above may be considered based on the preferences of the students. A project consists of choosing a paper that uses category theory and matches the research interests of a student, writing a short note about it, and presenting it in class.

5 Notes

Notes will be posted on the course website. These notes may not provide sufficient level of detail, so students should still take their own notes.

6 Announcements

Announcements about homework and other matters will be made via TTU email. Students are expected to check their TTU email regularly for updates.

7 Schedule

The following schedule may be subject to change as the semester progresses.

There will be a total of 43 lectures on the following days:

1/19: Introduction. Categories: definition, sets, vector spaces.

1/22: Algebraic examples. General topology. Functional analysis.

1/24: Differential operators as a category.

1/26: Isomorphisms. Groupoids. The fundamental groupoid.

1/29: The absolute Galois groupoid. Functors: definition, forgetful functors, free group, dual vector space.

1/31: The opposite category. Contravariant functors. Examples.

2/2: The Hahn-Banach theorem as an equivalence of categories.

2/5: Concrete categories. Monomorphisms and epimorphisms.

2/7: Examples of monomorphisms and epimorphisms.

2/9: Equivalences of categories: motivation.

2/12: Equivalences of categories: definition. Example: duality on vector spaces.

2/14: The Hahn-Banach theorem.

2/16: Equivalences are precisely fully faithful essentially surjective functors.

2/19: Pontryagin duality.

2/21: C*-algebras and Gelfand duality.

2/23: Von Neumann algebras, Borel functional calculus, spectral theorem.

2/26: Natural transformations.

2/28: The Riesz representation theorem.

3/2: Covering spaces and the fiber functor.

3/5: Sheaves of sets on topological spaces.

3/7: Equivalence between sheaves and etale spaces.

3/9: Galois theory of covering spaces.

3/19: Products. Products of sets.

3/21: Products of topological spaces.

3/23: Products of Banach spaces.

3/26: Coproducts.

3/28: The sheaf of measurable functions.

3/30: Examples of coproducts.

4/4: Equalizers.

4/6: Coequalizers.

4/9: Coequalizers of groups. Sequential colimits.

4/11: Sequential limits. Compactly supported smooth functions.

4/13: Limits. Pullbacks.

4/16: Pushouts. Limits via equalizers and pullbacks.

4/18: Filtered colimits and compact objects.

4/20: Corepresentable functors. Tensor products of vector spaces.

4/23: Representable functors. Exponentiable topological spaces.

4/25: Grasmannians as representable functors.

4/27: The Yoneda lemma. Presheaves as generalized objects. Smooth spaces.

4/30

5/2

5/4

5/7

8 Operating policy 34.04, §4: Class attendance

Responsibility for class attendance rests with the student. Regular and punctual attendance at all scheduled classes is expected, and the university reserves the right to deal at any time with individual cases of non-attendance.

The instructor determines the effect of absences on grades consistent with university policy for excused and unexcused absences. When absences jeopardize a student's standing in a class, it is the responsibility of the instructor to report that fact to the student's dean. Excessive absences constitute cause for dropping a student from class. The drop may be initiated by the instructor but must be formally executed by the academic dean. If the drop occurs before the 45th class day of a long semester or the 15th class day of a summer term, the Office of the Registrar will assign a grade of DG. If the drop occurs after those times, the student will receive an F. In extreme cases, the academic dean may suspend the student from the university.

Department chairpersons, directors, or others responsible for a student representing the university on officially approved trips must notify the student's instructors of the departure and return schedules. The instructor so notified must not penalize the student, although the student is responsible for material missed. Any student absent because of university business must be allowed to make up missed work within a reasonable span of time or have alternate grades substituted for work due to an excused absence. Students absent because of university business must be given the same privileges as other students; e.g., if other students are given the choice of dropping one of four tests, then students with excused absences must be given the same privilege.

In case of an illness that will require an absence from class for more than one week, the student should notify her/his academic dean. The dean's office will inform the student's instructors through the departmental office. In case of class absences because of a brief illness, the student should inform the instructor directly.

Refer to OP 34.19, Student Absence for Observance of Religious Holy Days, for information regarding an absence to observe a religious holy day.

9 Operating policy 34.19: Student absence for observance of religious holy day

1. "Religious holy day" means a holy day observed by a religion whose places of worship are exempt from property taxation under Texas Tax Code §11.20.

2. A student who intends to observe a religious holy day should make that intention known in writing to the instructor prior to the absence. A student who is absent from classes for the observance of a religious holy day shall be allowed to take an examination or complete an assignment scheduled for that day within a reasonable time after the absence.

3. A student who is excused under section 2 may not be penalized for the absence; however, the instructor may respond appropriately if the student fails to complete the assignment satisfactorily.

10 Operating policy 34.22, §2b: Reasonable accommodation for students with disabilities

Any student who, because of a disability, may require special arrangements in order to meet the course requirements should contact the instructor as soon as possible to make any necessary arrangements. Students should present appropriate verification from Student Disability Services during the instructor's office hours. Please note: instructors are not allowed to provide classroom accommodations to a student until appropriate verification from Student Disability Services has been provided. For additional information, please contact Student Disability Services in West Hall or call 806–742–2405.

11 Operating policy 34.12, §5: Academic dishonesty definitions

Students must understand the principles of academic integrity, and abide by them in all class and/or course work at the University. Academic Misconduct violations are outlined Part I, section B.1 of the Code of Student Conduct. If there are questions of interpretation of academic integrity policies or about what might constitute an academic integrity violation, students are responsible for seeking guidance from the faculty member teaching the course in question.

Academic misconduct includes cheating, plagiarism, collusion, falsifying academic records, misrepresenting facts, violations of published professional ethics/standards, and any act or attempted act designed to give unfair academic advantage to oneself or another student. Additional information about academic misconduct is available in the Texas Tech University Handbook in Part II, section B of the Community Policies section in the Student Handbook at http://www.depts.ttu.edu/dos/handbook/.

12 Civility in the classroom

Texas Tech University endeavors to foster a classroom climate of mutual respect among students and between students and teacher. Mutual respect means that we should be tolerant of different ideas and varying opinions about topics of discussion in class, that we address each other respectfully and without interrupting while others are speaking, and that we do not engage in disruptive behavior in class. Signs of disrespect include, but are not restricted to: using cell phones (students must turn them off or leave them at home), reading a newspaper or other material that is not part of a class assignment while in class, talking with classmates during class, eating in class, and similar disruptive behaviors. Students who engage in disruptive behavior will be warned. Repeated disruptive behavior may result in the student being asked to leave the classroom.