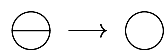
Mathematics 5324 (Topology)

Midterm 2

1. The *orange peel* with c carpels ($c \geq 1$) is a simplicial set O_c generated by the following system of generators and relations. There are two generating 0-simplices, N and S (north and south pole). There are c generating 1-simplices e_i, indexed by elements of $\mathbf{Z}/c\mathbf{Z}$. We have $\mathrm{d}_0(e_i) = S$ and $\mathrm{d}_1(e_i) = N$, i.e., each generating 1-simplex goes from the north pole to the south pole, just like one would cut an orange. There are c generating 2-simplices f_i, also indexed by elements of $\mathbf{Z}/c\mathbf{Z}$. We have $\mathrm{d}_0(f_i) = \mathrm{s}_0(S)$, $\mathrm{d}_1(f_i) = e_{i+1}$, and $\mathrm{d}_2(f_i) = e_i$, i.e., the 2-simplex f_i looks like a bigon squeezed between e_i and e_{i+1}, with one of the edges collapsed to a point.

- Draw a picture of O_1, O_2, and O_3.
- Compute the homology of O_c for all $c \geq 1$. (If an arbitrary c presents insurmountable difficulties, take $c = 3$, which will be valued almost as much.)
- Given $c \geq 1$ and $d \geq 1$, consider the homomorphism of abelian groups $r_d\colon \mathbf{Z}/dc\mathbf{Z} \to \mathbf{Z}/c\mathbf{Z}$ that sends an element $x = [u] \in \mathbf{Z}/dc\mathbf{Z}$ to the element $[u] \in \mathbf{Z}/c\mathbf{Z}$ represented by the same integer u. Construct a simplicial map $O_{dc} \to O_c$ that sends $S \mapsto S$, $N \mapsto N$, and $f_i \mapsto f_{r_d(i)}$.
- Compute the homology of the simplicial map $O_{dc} \to O_c$. (If arbitrary c and d present insurmountable difficulties, take $d = 3$ and $c = 1$, which will be valued almost as much.)

2. Triangulate and compute the homology of the following map:



The outer circle is mapped via the identity map, whereas the middle bar is projected onto the lower semicircle (so that its endpoints do not move).

3. Compute the homology of simplicial set with a single generating simplex σ of dimension 3 with relations $\mathrm{d}_0(\sigma) = \mathrm{d}_1(\sigma) = \mathrm{d}_2(\sigma) = \mathrm{d}_3(\sigma)$.

4. Construct *all* possible simplicial maps $\mathrm{S}^{\mathbf{2}} \to \mathrm{S}^{\mathbf{2}}$ and compute their homology. Recall that $\mathrm{S}^{\mathbf{2}}$ is a simplicial set generated by a single 2-simplex σ and a single 0-simplex $*$ with relations $\mathrm{d}_i(\sigma) = \mathrm{s}_0(*)$ for all $i \in \{0, 1, 2\}$.