Qualifying Exam Syllabus for Dmitri Pavlov (pavlov@math)

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Major topic: Algebraic topology (Geometry)

Ordinary homology and cohomology: Standard exact sequences, cup and cap products, universal coefficients, Künneth formula, transfer homomorphism, local coefficients, Poincaré duality, Alexander duality, Bockstein homomorphisms for $\mathbf{Z}/(p)$, Lefschetz fixed point theorem, H-spaces, Hopf algebras, and cohomology of SO.

Homotopy: Whitehead theorem, Eilenberg-Mac Lane spaces, Hurewicz theorem, CW approximations, excision (with additional conditions) for homotopy groups and Freudenthal suspension theorem, fibrations, Whitehead products and stable homotopy groups, Postnikov tower, obstruction theory, loopspace and suspension functors, Leray-Hirsch theorem, cohomology operations, Steenrod squares and powers, cohomology of U, SU, and Sp, cohomology of Grassmannians, Gysin sequence, Thom class, Thom space, and Thom isomorphism, Eckmann-Hilton duality, Dold-Thom theorem.

Cobordism and stable algebraic topology: Unreduced and reduced cohomology theories, uniqueness of ordinary cohomology, Brown representability theorem, multiplicative structure. Leray-Hirsch theorem, complex-oriented cohomology theories, splitting principle, Chern class and cohomology of BU, Thom isomorphism, complex cobordism and Thom class, Gysin sequence. Stable homotopy groups as reduced homology theory. Basic properties of complex cobordism, formal group laws, and related cohomology theories.

Major topic: Complex geometry (Geometry)

Basic definitions, divisors and line bundles.
Kähler identities, Hodge decomposition, Hard Lefschetz theorem, Hodge conjecture.
Serre duality, connections, curvature, Chern classes and Chern-Weil theory.
Kodaira vanishing theorem, Kodaira embedding theorem.
Deformation of complex structures, Kodaira-Spencer theorem, Kuranishi theorem. (Without proofs.)

Minor topic: von Neumann algebras (Modern analysis)

GNS construction.
Definition of von Neumann algebras.
von Neumann bicommutant theorem.
Kaplansky density theorem.
Commutative von Neumann algebras.
Geometry of projections in a factor and types of factors.
The trace and the standard form of type II_1 factor.

References

[1] A. Hatcher, Algebraic Topology.
[2] A. Kono, D. Tamaki, Generalized Cohomology.
[3] D. Huybrechts, Complex Geometry.
[4] V. Jones, Von Neumann Algebras.
[5] M. Takesaki, Theory of Operator Algebras I.